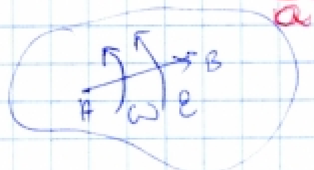
Mechanika ogólna II
- kinematyka zajęcie 3.

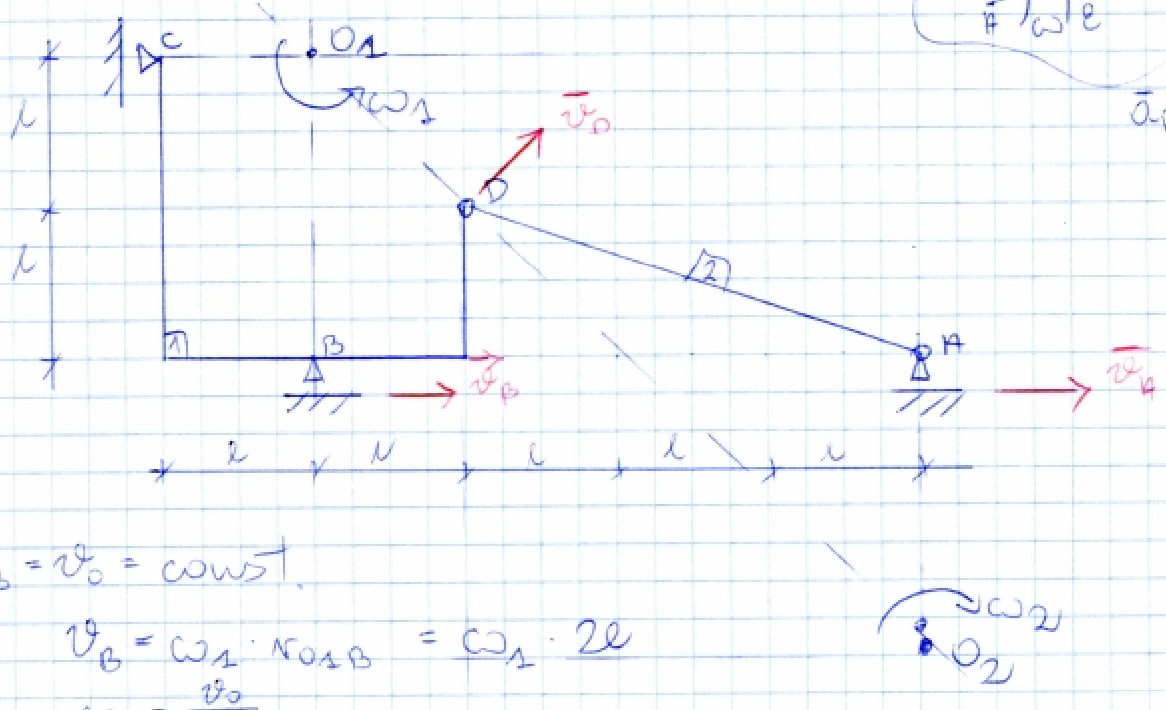
$\bar{a}^n = \omega^2 \cdot r_{AB}$

$\bar{a}^\tau = \varepsilon \cdot r_{AB}$



$\bar{a}_B = \bar{a}_A + \bar{a}_{B|A}$

Zadanie 1. $\bar{v}_A = ?$, $\bar{a}_A = ?$



$v_B = v_0 = const.$

$v_B = \omega_1 \cdot r_{O_1B} = \omega_1 \cdot 2l$

$\omega_1 = \frac{v_0}{2l}$

$v_D = \omega_1 \cdot r_{O_1D} = \frac{v_0}{\cancel{2}l} \cdot \sqrt{2}\, l = \frac{\sqrt{2}}{2} v_0$

$v_D = \omega_2 \cdot r_{O_2D} = \omega_2 \cdot 3\sqrt{2}\, l$

$\frac{\cancel{\sqrt{2}}}{2} \cdot v_0 = \omega_2 \cdot 3\cancel{\sqrt{2}}\, l$

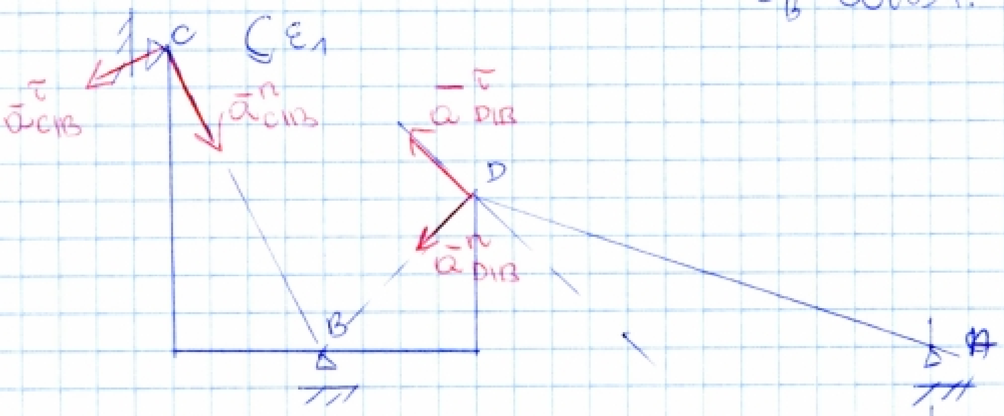
$\omega_2 = \frac{1}{6} \frac{v_0}{l}$

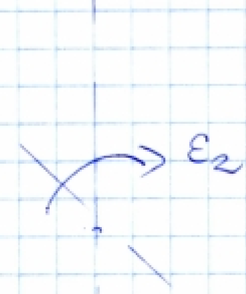
$v_A = \omega_2 \cdot r_{O_2A} = \frac{1}{6} \cdot \frac{v_0}{\cancel{l}} \cdot 2\cancel{l} = \frac{1}{3} v_0$

$v_{Ax} = \frac{1}{3} v_0$

$v_{Ay} = 0$

$v_B = const. \Rightarrow a_B = 0$





(1) $\bar a_C = \bar a_B + \bar a_{C|B} = \bar a_B + \bar a^n_{C|B} + \bar a^\tau_{C|B}$

(2) $a_{Cx} = 0$

(3) $\bar a^n_{C|B} = \omega_1^2 \cdot r_{CB} = \frac{v_0^2}{4l^2} \cdot \sqrt{5}\, l = \frac{\sqrt{5}}{4}\frac{v_0^2}{l}$

(4) $\bar a^\tau_{C|B} = \varepsilon_1 \cdot r_{CB} = \varepsilon_1 \sqrt{5}\, l$

rzutując (1) na Ox i z (2) mamy

$$0 = \frac{1}{\sqrt{5}}\bar a^n_{C|B} - \frac{2}{\sqrt{5}}\bar a^\tau_{C|B} = \frac{1}{\cancel{\sqrt{5}}}\cdot\frac{\cancel{\sqrt{5}}}{4}\cdot\frac{v_0^2}{l} - \frac{2}{\cancel{\sqrt{5}}}\varepsilon_1\cancel{\sqrt{5}}\, l$$

$$\frac{v_0^2}{4l} = 2\varepsilon_1 \cdot l$$

(5) $\varepsilon_1 = \frac{1}{8}\frac{v_0^2}{l^2}$ – przyspieszenie kątow

(6) $\bar a_D = \bar a_B + \bar a^n_{D|B} + \bar a^\tau_{D|B}$

(7) $a^n_{D|B} = \omega_1^2 \cdot r_{BD} = \frac{v_0^2}{4l^2}\cdot\sqrt{2}\, l = \frac{\sqrt{2}}{4}\frac{v_0^2}{l}$

(8) $a^\tau_{D|B} = \varepsilon_1 \cdot r_{BD} = \frac{1}{8}\frac{v_0^2}{l^2}\cdot\sqrt{2}\, l = \frac{\sqrt{2}}{8}\frac{v_0^2}{l}$

(9) $\bar a_A = \bar a_D + \bar a^n_{A|D} + \bar a^\tau_{A|D} = \bar a^n_{D|B} + \bar a^\tau_{D|B} + \bar a^n_{A|D} + \bar a^\tau_{A|D}$

(10) $\bar a_{Ay} = 0$

(11) $a^n_{A|D} = \omega_2^2 \cdot \sqrt{10}\, l = \frac{v_0^2}{36 l^2}\cdot\sqrt{10}\, l = \frac{\sqrt{10}}{36}\frac{v_0^2}{l}$

(12) $a^\tau_{A|D} = \varepsilon_2 \cdot \sqrt{10}\, l$

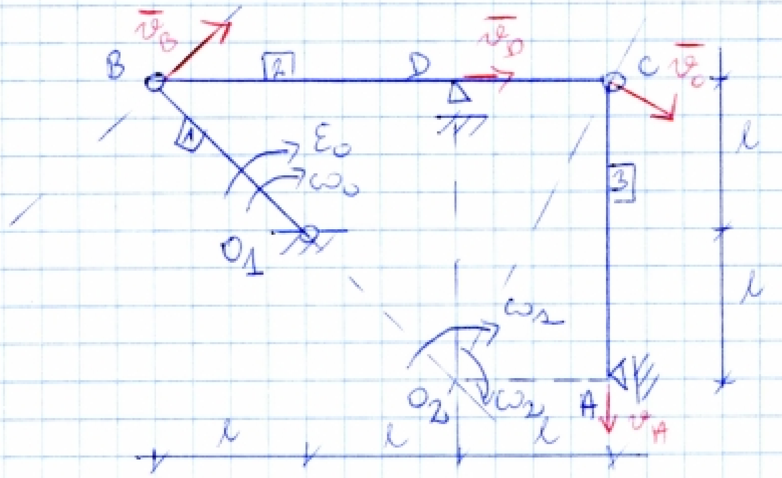
rzutując (9) na Oy i z (10) mamy

$$\varepsilon_2 = -\frac{7}{216}\frac{v_0^2}{l^2}$$

rzutując teraz (9) na Ox dostajemy

$$a_{Ax} = -\frac{23}{54}\frac{v_0^2}{l^2}$$

Zadanie 2. Znając ω_0 i ε_0 znajdź $\bar{v}_A$ i $\bar{a}_A$.



$$v_B = \sqrt{2}\,l\cdot\omega_0 = r_{O_1B}\cdot\omega_0 \quad \Rightarrow \quad \omega_1 2\sqrt{2}\cancel{l} = \sqrt{2}\cancel{l}\cdot\omega_0$$

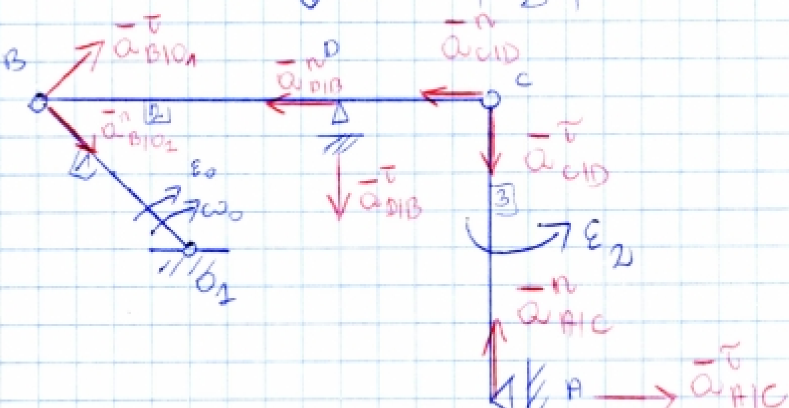
$$v_B = r_{O_2B}\cdot\omega_1 = 2\sqrt{2}\,l\,\omega_1 \qquad \omega_1 = \tfrac{1}{2}\omega_0$$

$$v_C = \omega_1\cdot\sqrt{5}\,l = \tfrac{\sqrt{5}}{2}\omega_0\cdot l = \omega_1\cdot r_{O_2C} \quad \Rightarrow \quad \omega_1 = \omega_2$$

$$v_C = \omega_2\cdot r_{O_1C}$$

$$v_A = \omega_2\cdot r_{O_2A} = \tfrac{1}{2}\omega_0 l$$

ε_1 - przyspieszenie tarczy 2



(1) $\bar{a}_B = \bar{a}_{O_1} + \bar{a}_{B|O_1} = 0 + \bar{a}^n_{B|O_1} + \bar{a}^\tau_{B|O_1} = \sqrt{2}\,l\cdot\omega_0^2 + \sqrt{2}\,l\cdot\varepsilon_0$

(2) $\bar{a}_D = \bar{a}_B + \bar{a}^n_{D|B} + \bar{a}^\tau_{D|B}$

(3) $\bar{a}_{Dy} = 0$

i stąd mamy

(4) $\bar{a}_{Dy} = 0 \overset{(1)}{\underset{(2)}{=}} -\frac{1}{\cancel{\sqrt{2}}}\cancel{\sqrt{2}}\cancel{l}\cdot\omega_0^2 + \frac{1}{\cancel{\sqrt{2}}}\cancel{\sqrt{2}}\cancel{l}\cdot\varepsilon_0 + 0 - 2\cancel{l}\cdot\varepsilon_1 = 0$

$$\varepsilon_1 = \tfrac{1}{2}\left(\varepsilon_0 - \omega_0^2\right)$$

(5) $\bar{a}_C = \bar{a}_D + \bar{a}^n_{C|D} + \bar{a}^\tau_{C|D}$

(6) $\bar{a}_A = \bar{a}_C + \bar{a}^{n}_{A|C} + \bar{a}^{\tau}_{A|C}$

zatem z (5) i (6)

(7) $\bar{a}_A = \bar{a}_D + \bar{a}^{n}_{C|D} + \bar{a}^{\tau}_{C|D} + \bar{a}^{n}_{A|C} + \bar{a}^{\tau}_{A|C}$

rzutując to na O_y otrzymujemy

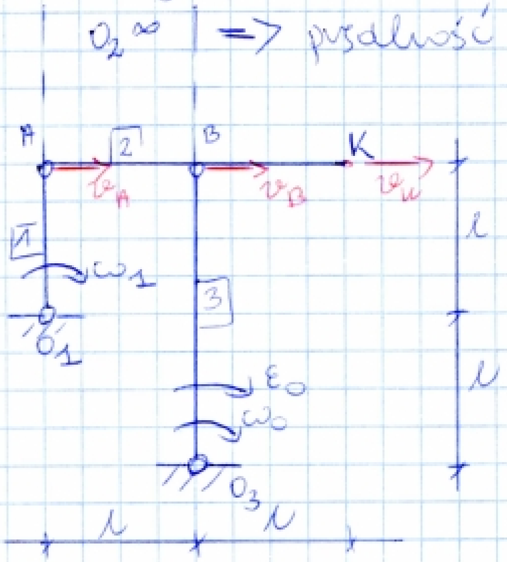
(8) $$a_{A_y} = 0 + 0 - \varepsilon_1 \cdot l + \omega_2^2 \cdot l + 0 =$$

$$= -\tfrac{1}{2}(\varepsilon_0 - \omega_0^2) l + \tfrac{1}{4}\omega_0^2 \cdot 2l =$$

$$= \omega_0^2 l - \tfrac{1}{2}\varepsilon_0 l = \tfrac{1}{2} l (2\omega_0^2 - \varepsilon_0)$$

Zadanie 3. Znając ω_0 i ε_0 znaleźć $\bar{v}_K$ i $\bar{a}_K$.

O_2^{∞} | => prędkość kątowa tarczy 2 $\omega_2 = 0$



Prędkości:

(1) $v_B = 2l \cdot \omega_0$ ponieważ

(2) $v_A = v_B$ i

(3) $v_A = \omega_1 \cdot r_{O_1A} = \omega_1 \cdot l$ mamy, że

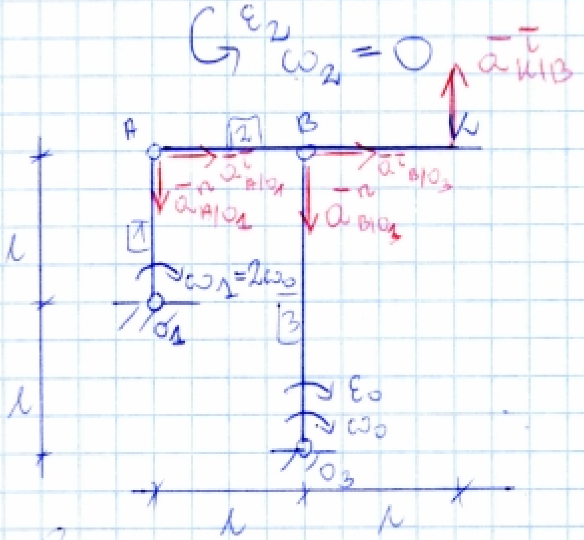
(4) $2l \cdot \omega_0 = \omega_1 \cdot l \Rightarrow \omega_1 = 2\omega_0$ i dalej

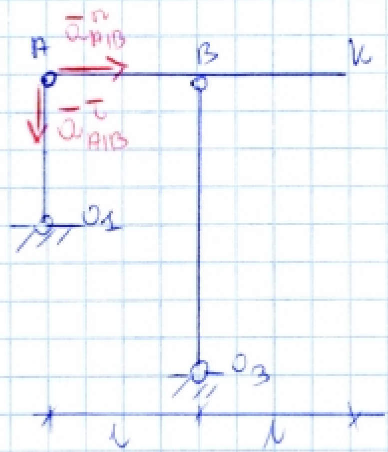
(5) $\bar{v}_K = \bar{v}_B + \bar{v}_{K|B} \Rightarrow v_{K|B} = \omega_2 \cdot l = 0$

ewentualnie z rzutu prędkości na oś przechodzącą przez B i K mamy, że $v_K = v_B$ czyli

$$v_K = 2\omega_0 l$$

Przyspieszenie:





$$\bar{a}_B = \cancel{\bar{a}_{O_3}}^{\,0} + \bar{a}^{n}_{B|O_3} + \bar{a}^{\tau}_{B|O_3}$$

$$\bar{a}_A = \cancel{\bar{a}_{O_1}}^{\,0} + \bar{a}^{n}_{A|O_1} + \bar{a}^{\tau}_{A|O_1}$$

(bo $\omega_2 = 0$)

$$\bar{a}_A = \bar{a}_B + \bar{a}^{n}_{A|B} + \bar{a}^{\tau}_{A|B} = \bar{a}^{n}_{B|O_3} + \bar{a}^{\tau}_{B|O_3} + \cancel{\bar{a}^{n}_{A|B}} + \bar{a}^{\tau}_{A|B}$$

$$\bar{a}^{n}_{A|O_1} + \bar{a}^{\tau}_{A|O_1} = \bar{a}^{n}_{B|O_3} + \bar{a}^{\tau}_{B|O_3} + \bar{a}^{\tau}_{A|B}$$

rzutując to na O_y mamy

$$-4\omega_0^2 \cdot l + 0 = -2\omega_0^2 \cdot l + 0 + \varepsilon_2 \cdot l$$

$$-2\omega_0^2 \cdot l = -\varepsilon_2 \cdot l$$

$$\varepsilon_2 = 2\omega_0^2$$

zatem (bo $\omega_2 = 0$)

$$\bar{a}_K = \bar{a}_B + \cancel{\bar{a}^n_{K|B}} + \bar{a}^\tau_{K|B} = \bar{a}^n_{B|O_3} + \bar{a}^\tau_{B|O_3} + \bar{a}^\tau_{K|B}$$

$$\cancel{= 2\omega_0^2 l + 2\varepsilon_0 l}$$

$$a_{Kx} = a^\tau_{B|O_3} = 2\varepsilon_0 \cdot l$$

$$a_{Ky} = -\bar{a}^n_{B|O_3} + a^\tau_{K|B} = -2l\omega_0^2 + l \cdot 2\omega_0^2 = 0$$